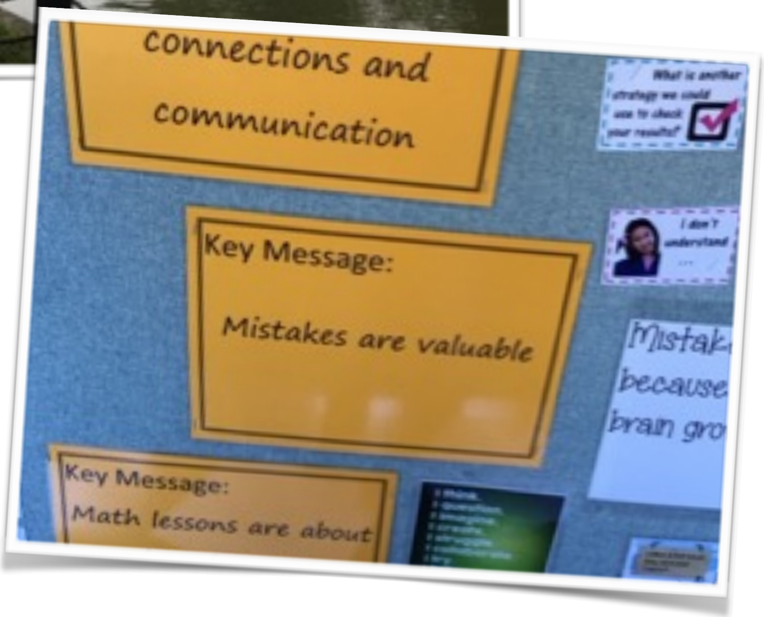
Sabbatical Report

Improving Mathematics in our Schools



Ideas to Challenge and Improve

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Improving the Teaching of Mathematics in Primary Schools

Acknowledgements

The more I researched this interesting topic the more questions it evoked in my mind. Where would it stop…it doesn't and nor it should as we continue to gain more insight into how the mind learns.

So many people have assisted me with this learning journey which was to:
"To raise student achievement and learning in mathematics for the students of Gordonton School through the principal researching best practice and exploring schools who have developed successful models and skills as part of the their high performing results."

I would like to thank the Board of Trustees of Gordonton School for their encouragement and support accorded me. Also to the management and staff of our school who continued to keep everything ticking over in my absence.
To Brenda Walker, Julie Smith from the Institute of Professional Learning*: Te Whai Toi Tangata*at of Waikato University who challenged my fixed mind-set and sent me on unknown pathways.

To the principals and staff of the schools from Auckland, Palmerston North, Hamilton Wanganui and New Plymouth who allowed me time to visit and talk with them and their staff. Also to the principals in Singapore whose school I visited; thank you for including me in your hectic daily schedules. I learned so much.

Sabbatical Intention

The intention of the sabbatical was to ultimately improve the mathematical ability of the students of Gordonton School through developing a way of teaching mathematics that enabled them to cope with an ever increasing numerical world. Along side this was to see if there was a way to achieve the above without increasing the work load of our teachers.

While our National Standard results in Mathematics have been encouraging they have not been as 'good' as those for writing or reading. Therefore what could we do to readdress this.

Background to Sabbatical

Late in 2014 we applied and were accepted into a mathematical professional learning community which began in 2015. While wanting a full scale investigation into mathematical practice at our school financially this wasn't an option at this time. So we settled with a very revised version which basically looked at what we were doing and how it could be improved.

During the first two terms of 2015 I worked with a group of 8 Year 7/8 students who had been identified as having sound mathematical knowledge and introduced them to Book 6 A of the Singapore Mathematics Curriculum. We worked our way through this text for about an hour a day, four days a week and what impressed me the most during these sessions were use of the visual models to assist in solving the variety of problems presented.

I was also reminded of the Lester Flockton booklet on Prime Mathematics Texts which on closer inspection were Scholastic's adaptation of the same Singapore Mathematics Texts to a New Zealand context.

The ten weeks of the sabbatical were spent researching the variety of articles provided by Brenda Walker which opened my mind to the world of:
Carol Dweck's Mind Set work
Jo Boaler', Unlocking Children's Math Potential , also her research on Speed, Making Mistakes and being Persistent. I watched her 68 videos on Making Sense with Mathematics
Sian Beilocks Choking Under Pressure
Paul Lockhart's Mathematical Lament
Glenda Anthony and Margaret Walshaw - Effective Pedagogy in Mathematics
Numerous websites - 'Whenmathhappens.com

During the sabbatical I visited schools throughout the North Island. There was always going to be an overseas component of this study, originally to Shanghai, however as it turned out Singapore was decided upon as it furthered my understanding of the extremely high world ranking in mathematics that the Island State holds.

My… How Things Change

Back in 2012 I wrote a paper for our staff outlining my mathematical beliefs especially why I believed the Numeracy Project had failed our students. It was based around the assessment practice more than anything else that you had to show a variety of strategies to solve a problem before you could advance to the next level. At this time I was convinced that we were overly confusing our students by providing multiple methods of solving equations and one go to strategy was all that was required. Hey it had worked for me!

Below are some of the things that I wrote.
"*Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals and percentages.*" Level 3 NZ Curriculum.

I believe that we are doing our students a huge disservice mathematically when we are assessing them on a 'range' of strategies to solve an equation. I believe that they only need one and if they only have one strategy and it gets the equation right then we/they have succeeded. They need at least ONE strategy that works, and it works in school and life situations. It may not be the quickest, it may not be the most efficient but if it works, it works.

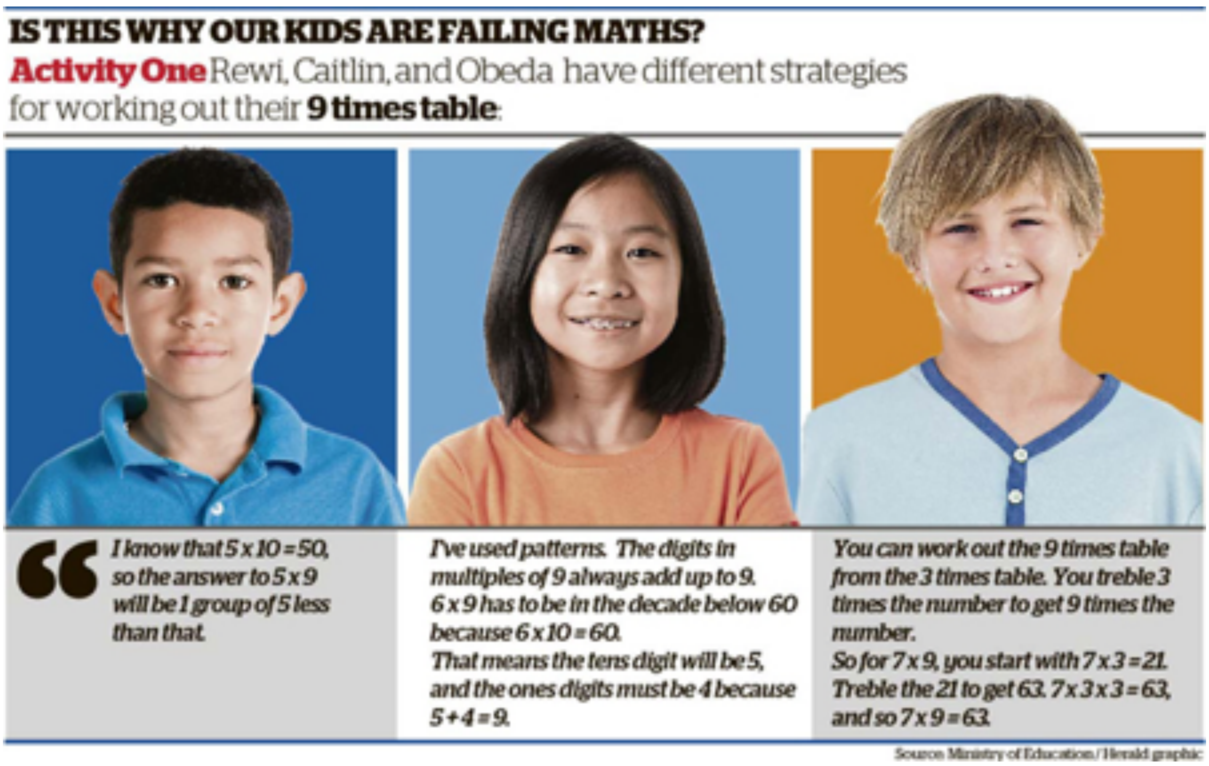
By all means, once they have this strategy in place, introduce them to other ways but they MUST have a solid grasp of ONE strategy before you hit the variation.

Mathematics is an exacting discipline – the answer is either right or it is wrong! Spelling too is an exacting discipline – the word is either spelt rightly or it is wrong. We don't assess how many ways a word can be spelt.

"Spell DUCK – d-u-c-k, now spell it using the consonants, spell it backwards, spell it using alternative letters."

In May/ June 2015 the New Zealand Herald published the following:
"Report finds Kiwi Kids are failing to grasp maths fundamentals
(May 30 2015)
The national study of about 4000 students found only 41 per cent of those at Year 8 - mostly aged 12 - met a Level 4 or higher standard of achievement in mathematics and statistics.

Back to basics call on maths
(June 4 2015)
Education Minister to launch report criticising primary teachers for letting children



down.

The paper, written by a researcher for the New Zealand Initiative (NZI) business group, criticises a $70 million Government maths project for failing to improve results and says teachers' maths abilities are letting children down.
"Too many children are not learning the basics off by heart at school. And, paradoxically, this is what is holding them back from developing a more complex understanding of maths," the report said.

I proudly produced my paper from a couple of years back and said loud and long to anyone who was old school enough to listen that, "See I was right!' Therefore I came into the sabbatical with a very fixed view of what was needed to improve mathematics. Thankfully I was sufficiently challenged and had my eyes opened enough to realised how wrong I was.

I began by reading the ideas of Carol Dweck on Mind-set and from here my work took off into places previously unknown. I was introduced to a vast array of mathematicians and not educational bureaucrats who challenged my thinking and thankfully reshaped it.
- Carol Deweck's Mind-set - fixed and open
- Jerome Bruner
- Mistakes and Persistence
- Heterogeneous v Ability grouping
- The fallacy of speed in mathematics - maths is a slow contemplative process
- Changing the language - right and wrong
- The importance of visualisation
- Collaboration
- Number Sense
- Messages we give students
- Acceleration v remedial work

Wonderings-
- Is teacher lack of knowledge a concern.
- Would a text approach a-la- Singapore help here
- Is there a problem with the text book approach

So What's the Buzz?

Now I know there are some fantastic teachers of mathematics, with creative student centred lessons and that there are also students in classrooms who 'hate' mathematics, who are dumb at it and find it mind numbingly boring.

Paul Lockhart in his "A Mathematician's Lament" looks at traditional mathematics teaching. I wonder where you sit? My gut feel is that much of our mathematics teaching isn't too far removed from how we were taught and while we are having a warm up, 3+ groups, games and activities, and a reflection, the way we teach is little changed.

…if I had to design a mechanism for the express purpose of destroying a child's natural curiosity and love of pattern-making, I couldn't possibly do as good a job as is currently being done— I simply wouldn't have the imagination to come up with the kind of senseless, soul- crushing ideas that constitute contemporary mathematics education. Paul Lockhart

Triangle Area Formula:
A = 1/2 b h h b
“The area of a triangle is equal to one-half its base times its height.” Students are asked to memorise this formula and then “apply” it over and over in the “exercises.” Gone is the thrill, the joy, even the pain and frustration of the creative act. There is not even a problem anymore. The question has been asked and answered at the same time— there is nothing left for the student to do….By removing the creative process and leaving only the results of that process, you virtually guarantee that no one will have any real engagement with the subject. Paul Lockhart

By concentrating on what, and leaving out why, mathematics is reduced to an empty shell. The art is not in the “truth” but in the explanation, the argument. It is the argument itself which gives the truth its context, and determines what is really being said and meant. Mathematics is the art of explanation. If you deny students the opportunity to engage in this activity— to pose their own problems, make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs— you deny them mathematics itself. So no, I’m not complaining about the presence of facts and formulas in our mathematics classes, I’m complaining about the lack of mathematics in our mathematics classes. Paul Lockhart

By concentrating on what, and leaving out why, mathematics is reduced to an empty shell

To do mathematics is to engage in an act of discovery and conjecture, intuition and inspiration; to be in a state of confusion— not because it makes no sense to you, but because you gave it sense and you still don’t understand what your creation is up to; to have a breakthrough idea; to be frustrated as an artist; to be awed and overwhelmed by an almost painful beauty; to be alive, damn it. Remove this from mathematics and you can have all the conferences you like; it won’t matter. Operate all you want, doctors: your patient is already dead.

If teaching is reduced to mere data transmission, if there is no sharing of excitement and wonder, if teachers themselves are passive recipients of information and not creators of new ideas, what hope is there for their students? If adding fractions is to the teacher an arbitrary set of rules, and not the outcome of a creative process and the result of aesthetic choices and desires, then of course it will feel that way to the poor students.

We learn things because they interest us now, not because they might be useful later. But this is exactly what we are asking children to do with math.

But isn’t one of the purposes of mathematics education to help students think in a more precise and logical way, and to develop their “quantitative reasoning skills?” Don’t all of these definitions and formulas sharpen the minds of our students?
No they don’t. If anything, the current system has the opposite effect of dulling the mind. Mental acuity of any kind comes from solving problems yourself, not from being told how to solve them.

Mathematics is about problems, and problems must be made the focus of a students mathematical life. Painful and creatively frustrating as it may be, students and their teachers should at all times be engaged in the process— having ideas, not having *ideas, discovering patterns, making conjectures, constructing examples and counterexamples, devising arguments, and critiquing each other's work. Specific techniques and methods will arise naturally out of this process, as they did historically: not isolated from, but organically connected to, and as an outgrowth of, their problem-background.*

Doing mathematics should always mean discovering patterns and crafting beautiful and meaningful explanations.

So in Lockhart's mind we have gone about mathematics the wrong way. We have taught rules and formulas, in short we have taught the how not the why. Mathematics has been taught as an exacting discipline not as he believes as an art. This has lead to many like me who know the formulas and can use them but have no idea why they work or why I'm doing it.

Division of Fractions - invert the divisor and multiply

Maths is something that you do not something that is creative and enjoyable. Unfortunately an overriding reason for this is likely to be time! Time to cover everything that is expected of you in the very short amount of it that you don't have. So what to do?? I know just teach the formulas. They get the answers BUT not the understanding

We've gone wrong and it is time get it right. Where to begin?

What's Mind-set Got To Do With It?

Mind-set is a concept discovered and developed by Stanford University psychologist Professor Carol Dweck and published in her 2006 book Mind-set: The New Psychology of Success.

Briefly Dweck saw students and teachers as having two distinct mind-set that affected every part of their thinking and lives. Your mind-set affects your thinking about how you see yourself, the world and how you react to it.
" They may have a fixed mind-set in which they believe that intelligence is a static trait: some students are smart and some are not and that is that. Or a growth mind-set in which they believe intelligence can be developed by various means - by effort and instruction."

Dweck found that students who had a growth mind-set significantly outperformed their classmates who held a fixed mind-set. Why? Because they focused on learning, believing their effort and mistakes made them persistent and resilient when faced with set backs. *"Having a growth mind-set, with its emphasis on hard work in the service of learning, led to higher grades than a fixed mind-set, with all of the worries and defences that deflect students from applying themselves."*

Growth mind-set students were taught that their brain was like a muscle and that the more they used it the stronger and more effective it became. Also that every time they challenged themselves to learn something new their brain formed new connections and overtime they could be smarter.

The chart below gives a brief outline of the key messages.

	Fixed Mind-set	Growth Mind-set
	Intelligence is static Leads to a desire to look smart and therefore a tendency to..	Intelligence can be developed Leads to a desire to learn and a tendency to…
Challenge	Avoids challenges	Embrace challenges
Obstacle	Gives up easily	Persist in the face of setbacks
Effort	Sees effort as fruitless	Sees effort as the path to mastery
Criticism	Ignores useful or negative feedback	Learns from criticism
Success of Others	Feels threatened by others successes	Finds lessons and inspiration in the success of others
As a Result	They may plateau earlier and achieve less than their full potential	They reach even higher levels of achievement
All This	Confirms a deterministic view of the world	Gives them a greater sense of free will

Like most teachers and parents we were led to believe that positive self esteem was the key to success. Somehow if we praised how smart/clever/good looking/ strong… etc then the child would believe this and success would be assured. And so we did!

What Dweck found was that praising a 3 year old for being smart predicts their mind-set 5 years later. Deck's message is to praise wisely in mathematics - the effort, the engagement, the strategies, the perseverance, their resilience and their improvement. We have tended to praise the 'getting it right' (right now) but we need to reward the process. " You are not their yet, but I believe in you and you will get there."

She has outlined some messages that promote a growth mind-set

- *We believe in your potential and are committed to helping everyone get smarter*
- *We value (and praise) taking on challenges, exerting effort and surmounting obstacles more than we value (and praise) natural talent and easy success.*
- *Working hard on new things makes you smarter - it makes your brain grow new connections.*
- *School is not a place that judges you. It is a place where people help you brain grown new connections.*

We have perpetuated the fixed mind-set in mathematics for far too long. The view that you either have or you haven't got a 'maths brain' has influenced not only what but also how we teach. It also has a huge impact on generations of students that have been through our doors and we still get parents who say, " Well I couldn't do maths so that will be why Charlie can't." This gives permission to the child to not try!

Rheinberg found that *when teachers had a fixed mind-set, the students who had entered their class as low achievers left as low achievers at the end of the year.* But with growth mind set teachers students who started as low achievers moved up to moderate or high achievers. The teachers not only believe that every student can learn but they are committed to make it happen.

When teachers decide that certain students are not capable they may not take steps to help them develop their potential. They will comfort students with the old adage, "N*ot everyone can be good at maths*." Contrast this with growth mind-set teachers who encourage the students to try harder, and gave them suggestions and learning strategies.

Teachers with a fixed mind-set believe that learning is the students responsibility. If the students don't have what it takes then hard luck. For a teacher with a growth mind-set learning is a collaboration in which the teacher has great responsibility.

To truly embrace the concept of growth mind-set we will have to make major changes to the way we operate mathematics in our schools -

- Convince teachers and parents of its value
- That making mistakes is an essential part of learning
- Ability groups will be replaced by heterogeneous groups
- That collaboration and discussion are essential to understanding
- Change the way we question and respond to questions
- Are you able to make sense of this?
- Understanding the process is more important than the product
- Aligning ERO fixed beliefs

One fear that I have with the concept of mind-set is that Dweck has developed a specific growth mind-set programme which, as you would expect, costs a considerable amount of money. Most schools in New Zealand will like the concept but hate the cost and do it on the cheap resulting in a half baked result and major disappointment.

Get Rid of the Ability Groups

An educational tradition that probably had its origin in the English class system was the tried and true ability groups - high, medium,low. Clearly it makes sense. You put students of like ability together because:

- Ability grouping increases student achievement by allowing teachers to focus instruction.
- Teaching a group of like-ability students allows teachers to adjust the pace of instruction to students' needs. For example, a teacher might instruct at a slower pace---providing more repetition and reinforcement---with a group of low-achieving students than he or she would with a group of high achievers.
- A group of high achievers might be given more opportunities for independent research and cooperative group discussion than a group of low achievers would be given
- high achievers might be asked to apply their skills to solving higher-level thinking problems too.

It strikes me that it is something that we do, and have done for a long time because we have, not through any researched reason. Jo Boaler tells us that:

“Researchers now understand that every child can achieve at the highest levels in maths at school, if they are given the opportunities. Genes play a minimal part in learning and achievement but are eclipsed by the learning opportunities that are provided to students.”

It seems that students who struggle with maths do not have less potential to learn, they have just had fewer opportunities than others. Therefore it is important that these students be given the opportunities and encouraged in schools so they are able to progress to high mathematics levels.

Jo Boaler talks about ability grouping being the number 1 cause of fixed mind-set. They set up fixed messages about mathematical ability and intelligence. Ability groups fixes in their minds that they are in the top group so they are clever at maths and vice versa.
When students in England were grouped by ability 1/3 were put in the wrong group, 88% of students put into groups at age 5 stay in this group for the rest of their schooling.

The National Research Council in the UK have said ‘Formal and Informal tracking by ability be eliminated. Alternative strategies should be used to ensure appropriately challenging instruction for students who vary widely in their skill development.

Why then shouldn’t we be grouping? Boaler and others have found that overall achievement is lower than mixed ability grouping for a number of reasons.

- The messages that they give to students - the fixed mind-set (in high groups students believe they have the maths gift, in low groups the what’s the point, I can’t)
- Teacher expectation - if I expect you to do well, generally you will.
- The work they are given -in low groups the work is generally too easy, and are never given access to the higher level content of other groups. We have given work that they can get right not necessarily be challenged. Students need to be challenged.
- Borderline casualties - misplaced students in the wrong groups.
- Middle group teaching - same students, same work, same pace.

Michelle Obama when talking to a group of student said; *“No one is born smart, no one is born knowing how to read. No one is born knowing how to do maths. No one is born knowing how to play the flute. All of that comes with a lot of hard work.”*

Boaler talks a lot about heterogeneous groups of students working on a collective problem and ensuring that each member of the group is able to make sense of the problem before moving on. I”ll expand on this later in Mathematical Thinking Process

The key is not separating the students into ability groups - it is the collaborative discussion around a problem by all abilities and making sense of it. This is the key!

Making Mistakes

Peter Sims - *Successful people make mistakes, learn from them and set up on a new path*."

Alina Tugend- Better By Mistake - The Unexpected Benefit from Being Wrong *writes,*

"There are no simple fixes, but there are ways all of us can shift our thinking about mistakes. Starting with our children, we can emphasise effort and de-emphasise results. We can appreciate that we -- and they -- can't be perfect, nor is it a goal we should aim for. We should strive to do our best, but if the prize is ever-elusive perfection, then the fear of failure will too often overshadow the willingness to experiment, take risks and challenge ourselves. We should be careful of the contradictory message that it's all right to make mistakes but not where it counts, and of unintentionally making assumptions about gender that reinforce stereotypes -- that girls can't handle criticism or that boys don't want to talk about their mishaps. We can create a community of like-minded friends who support us in our efforts not to succumb to the idea that all that matters is good grades and awards. Yes, our children need to succeed, but we have to know -- and repeat it to ourselves over and over and over -- that they also need to fail.

When a mistake is made in mathematics the student is often left with a bad feeling, a feeling of weakness of understanding, associated with a believe that "I'm dumb!" We have students who are afraid to make mistakes and when this happens they are afraid of trying something new or being creative, of thinking in a different way. Many too respond to unknown problems by asking the teacher rather than trying different solutions that might be wrong! As Boaler says " They are victims of excellence."

Carol Dweck - "Every time a student makes a mistake in mathematics they grow a new synapse. The brain spark doesn't happen when they get work correct."

So making mistakes is the most important and useful thing to be doing in mathematics. We should be fostering students to make mistakes and we should not be giving them work they can already answer. I am not suggesting that we perpetuate the mistakes which often happens with work sheets where a child has the concept misunderstand and keeps repeating it. What it is saying is that to learn we need to make mistakes and this must be seen as an important part of the learning process.That making mistakes are necessary before new learning and understanding can occur. As John Edwards says 'You must go into a pit of uncertainty and bewilderment before eventually you understand.' Unfortunately for many of our students they never come out of that pit!

The research on mindset and the importance of making mistakes strongly suggest that our mathematics environments need to be ones which students:

- are given open tasks and challenging work
- work that causes them to struggle
- work which makes them think
- work that they will make mistakes

Also where teachers support and reward students for making mistakes so that they will feel comfortable in doing so.

Dweck emphasises too the importance of teacher language during this time. The use of the word 'Yet' is seen as a very powerful one. 'You are not quite there yet. I believe that you are very close and you will get there." They are hopeful messages and indicate that you are still learning.She also sees the importance of keeping track of student progress and sharing this progress with them to show them that all their effort is paying off.

Because of 'time' and the closed mindset of being right we have tended to gloss over errors and haven't given them the feed back they deserve. We have to turn this around so mistakes become our friend. They shouldn't be condemned or fled from. They are a natural part of learning - they give clues to what the student has done incorrectly and what they might do next. The mantra should be "*Through mistakes and effort I can grow my brain*."

When students are getting things wrong they must be taught to stop and ask, "What is it I don't understand? Try to work out what is happening. Unfortunately many will say that they understand a concept when clearly they don't! Dweck suggests that you turn the conversation from where the child thinks you are evaluating whether they are good at a mathematical concept or not, to the reason that some can understand the concept is because they have had opportunities in their past to learn it, and you am giving you this opportunity now.

Mistakes need to be expected, respected and inspected

For far too long we have focused on the product not the process in mathematics and praised the right answer. **We have been hung up on the answer! It does beg the question though that when mathematics is tested we look for the answer not the process.**

Persistence

The act of sticking with it until you are satisfied that you have achieved the goal.

In mathematics it is much easier to be spoon fed, because answers don't require thought. Guy Brousseau has come up with the teacher's dilemma which he calls The Didactic Contract. Basically it is:

- The student gets stuck
- They want help step by step to solve the problem
- They want to be led
- The teacher's role is to help so they help by leading.

This is then the traditional approach to helping students. However the problem with this is that it takes the 'thinking' the cognitive demand out of the equation and also means the student doesn't have to struggle.

To ensure we get a more persistent student we need to create mathematical problems that are open and challenging. The students must be comfortable in making mistakes because if they are uncomfortable with mistakes their confidence will be affected to the point where they will give up.

Why do we feel bad when we make mistakes? Two reasons:

1. It shows weakness in our understanding

2. It shows we are dumb

Jo Boaler suggest the testing culture is a major contributor to this. Students see success as doing well in a test and if they believe in this then mistakes don't play a role. Alina Tugend says;"*If students are afraid of mistakes then they are afraid of trying something new or being creative, or thinking in a different way. They're scared to raise their hands when they don't know the answer and their response to a difficult problem is to ask the teacher rather than try different solutions that might (gasp) be wrong*!" They are victims of excellence.

We must value mistakes and persistence!

The Fallacy of Speed in Mathematics

Mathematics should never be associated with speed!
Unfortunately we have for too long valued speed in mathematics. Remember all of those times table tests that we rigorously administered week in, week out with the belief that a good mathematician was fast at computation. Somehow speed = good, slow = not good! Many students see 'fast students' as those who are the best mathematicians, those who have the most potential. Laurent Schwartz believes that high level mathematical thinking is about working in depth not working at speed

Neuroscience now shows that **mathematics should never be associated with speed**.While timed test (Ikan) were used for the best intentions they now show that they at best led to mathematics anxiety and damage.

Sian Beilock and her colleagues conducted brain scans and found that maths facts, the ones we use in timed tests, are held in working memory. The more working memory and individual has the greater is their potential for academic success. However what she found was that stress blocks working memory and therefore blocks facts that they are familiar with. This is the familiar memory blank we have all experienced when we know the answer. It is stress blocking it and it affects all students on the learning/academic spectrum.

Laurent Schwartz a word renown French mathematician, who on his own admission was slow at maths says in his book A Mathematician Grappling with his Century:
" *Rapidity doesn't have a precise relationship with intelligence. What is important is to have a deep understanding of things and their relationship to each other. This is where intelligence lies. The fact of being quick or slow isn't really relevant. Naturally it helps to be quick, like having a good memory but it is neither necessary or sufficient for intellectual success.*"

So time/speed tests misrepresent mathematics as a subject about working on factual questions fast but this is not the mathematics mathematicians work on or is needed in the workplace. It reinforces the belief in students that only the best can do it quickly and it puts others off who think more slowly and deeply.

How quickly one can solve a problem does not have a precise relationship with intelligence. Intelligence lies in having a deep understanding of the pieces of the problem and their relationship to each other.

Being quick or slow isn't really pertinent. Speed helps, like having a good working memory helps, but it is neither necessary or an indicator of intellectual success. Speed tests misrepresent mathematics as a subject about working on questions fast but this isn't the mathematics that is necessary needed in the work place.

For many of our students speed is the enemy and should be used in moderation or not at all.

Teaching for a Growth Mind-set

To do justice to the whole concept of Mind-Set it needs to be taught correctly and this immediately creates a problem for schools in New Zealand. The cost for our school would be around $6000 (US) of $9,700. I note that an approved Mind-set trainer from Australia is available for professional learning which might work out cheaper.

However like much of what we do, cost makes us improvise and there are guidelines to teach for a growth mind set rather than what many of us are currently and traditionally taught for fixed mind-set. Teacher mind-set messages come from; grouping students, tasks worked on, assessment and grading, how mistakes are managed and the norms set in the classrooms.

1.Mind-Set Norms

It is important to set up norms around a growth mind-set. Cathy Humphreys suggests that we should be encouraging our students to be "*sense makers*." They need to be able to draw and visualise to make sense, they work in a collaborative classroom where ideas are readily shared (making sense when it is verbalised) where public space is given to making sense (whole class discussions) and that mistakes once again are valued and shared. "We need to build classrooms where mistakes or wrong answers are seen as useful." Classrooms where the new practice is not only to convince yourself of an answer, but you need to be able to convince a friend and a sceptic.

Such a strategy moves discussion away from an emphasis on whether the answer is right or wrong to whether the explanation is clear and convincing. Try for example trying to make sense of 1 ÷ 2/3. Find a visual that will help, go through your explanation and don't use the formula as we don't understand it yet.

2. Tasks

Most mathematical tasks that we set are short and require one answer so the students will either get it right or wrong. These tasks lead students to believe that mathematics is about performance and performing and not about learning and growing. *"Maths is too much answer time and not enough learning time."*

Obviously we need to change the way we question in mathematics and so it changes what the students are doing. For example-

From find the perimeter of a rectangle whose length is 10cm and its width is 3cm.

To construct two different rectangles with a perimeter of 30cm

The second question involves a different kind of thinking and provides the students with choices even though you are assessing the same mathematics. You are moving from performing to growing and thinking.
Change the dialogue from "*I'm finding the answer to I"m thinking and learning*."

3. The Growth Mind-Set Task Framework
At Stanford University Jo Boaler and colleagues have developed a framework for what growth mind-set tasks should look like:

- Openness
- Different ways of seeing
- Multiple end points
- Multiple paths/strategies
- Clear learning goals and opportunities of feedback

This framework leads to different forms of engagement, different learning opportunities. For example a closed example would by to solve 1 ÷ 2/3. A more growth mind-set task would be, “ What are the different ways you can make sense of 1 ÷ 2/3?”

4. Assessment Grading and Feedback
Most mathematics assessment are generally closed tools that tell students how well or badly they have performed in relation to other students and give them no idea about the path for growth and improvement. They do tend to be summative, taken at the end of a unit and show performance.

Assessment for learning is based on the principle that students have a clear sense of where they are going and what they need and have to do to be successful. Often students don’t know where they are at in their learning path, what they are learning, why it is being taught, and how they can be successful.

Assessment for learning is

Where the student is
△
Ways to close the gap
▽
Where they need to be

Most mathematics assessment give students a result, a ranking and can constantly knock their self esteem. That weekly tables test that only reinforces that right now I can’t do this especially in the time that you give me. Once you give a mark/grade that’s all the student looks at!

Butler did some research on the conditions or assessment. He had students given grades only and another group given diagnostic feedback and not surprisingly those who were given the feedback did significantly better in the future than the grades only students. Feedback helps students understand how they can and will improve.

Once again I wonder whether we test in this way simply because we have always done it and it is the quickest way! Test, mark, result, move on. Research suggests that providing students with feedback helps them understand how they will improve and teachers setting goals where the important ideas are expressed and show how they are linked.

“I understand the difference between mean and medium and know when and how they should be used.”

“ *I understand that fractional parts of a whole always have to be equal in size but not in shape.*”

Students know what is required and can assess themselves and their peers against this statement. They take greater responsibility for their own learning.

White and Fredrickson found that low achievers, who are thought to be lacking in ability or just slow, are low often because they don’t know what is important and what they are meant to be paying attention to. And that given the opportunity to discuss and make sense of questions greatly affects their ability to succeed.

5. Grouping

Ability grouping is thought to be the number 1 cause of a fixed mind-set. The biggest fixed mind-set message we can give to students is that they are high, low or middle ability students. Students instinctively know from how they are grouped how they are perceived. “I’m a Lightning McQueen and I’m a nothing.” When they are ability grouped within or across classes they develop fixed messages about their potential their ability and their intelligence.

It also affects those in the ‘top’ groups who see themselves as the smart ones and it is damaging for them because when they actually strike something that they don’t understand they are very likely to leave it and not attempt it. Why? Because they are meant to be smart and smart students know how to do things.

The countries that group their students for mathematics the least and the latest are the most successful in the world - Finland, Japan and Korea . In the England about 1/3 of the students that are grouped are put in the wrong group. Also 88% of students put into groups at age 5 stay in that group for the rest of their schooling.

Lisa Yiu observed and explained why Japanese teachers do not use ability grouping. *“In Japan what is important is balance. Everyone can do everything, we think that is a good thing, so we cannot divide by ability. Japanese education emphasises group education, not individual education.Because we want everyone to improve promote and achieve goals together, rather than individually. That’s why we want students to help each other, to learn from each other and to get along and grow together - mentally physically and intellectually.”* This last sentence is part of the reason for their high achievement in mathematics.

The National Research Council in the United States recently issued the following statement. “ ***Formal and informal grouping by ability be eliminated… Alternative strategies should be used to ensure appropriately challenging instruction for students who vary widely in their skill level.***”

So the question become why does grouping students in schools particular in mathematics result in lower overall achievement?

1. The messages they give to students - the fixed mind-set message that they are, or they aren't a mathematics person. In high groups the message is fixed that they have the 'maths gift.' In low groups the message is that they will struggle and get the least exciting type of mathematics
2. Teacher expectations - If we expect the children will be 'good at maths' then that is the way we teach them and often that is what we get.
3. The work they are given - it is often too easy, it limits their achievement and don't get access to high level content that others do. We have given work to low ability students that they will get correct and feel good about but doesn't really challenge them.
4. Borderline Casualties - these are the misplaced students.
5. Middle Group Teaching - giving all of the students the same work at the same pace.

Michelle Obama says. *"Because the truth is… and this is important. I want you all to listen up … no one is born smart. Do you understand that? No one is born smart. No one is born knowing how to read, right? No one is born knowing how to do math, or no one is born knowing how to play the flute… all of that comes with a lot of hard work.*

Changing to heterogeneous grouping will require a mammoth change in the mind-set of both teacher and parents and will take a monumental effort for it to occur as it is so *fixed* in New Zealand schools. Once again I wonder whether we have done this because logically it makes sense to teach to the learning similar needs of the group rather than as science is telling us now that such grouping causes huge mind-set damage.

6. Messages

We all - parents and teachers give mind-set messages to students. Bowler says that one of the most powerful messages we can give students as feedback is, "*I believe in you… you are not there yet.*"

Carol Dweck again tells us about the difference between a fixed and a growth mind-set.

Fixed Effort + Difficulty = dumb

Growth Effort + Difficulty = getting smarter

That through effort, making mistakes, making new neurological connections the brain grows. The problem below, if you don't solve it in one hit will illustrate this point.

Have a go at this Primary Level 6 Ratio problem from Singapore. Try and make sense of it yourself and if you aren't getting anywhere try it with colleagues. Where possible use visuals, perhaps the model approach mentioned below.

" *The ratio of the number of boys to the number of girls at a playground was 5:7*

After 7/10 of the boys and 1/4 of the girls left there were 54 children left at the playground.
How many children were there at the playground at first?"

Dweck strongly believes that the 'self esteem movement' was wrong and that telling students that they were intelligent, they were bright and talented was harmful and created a fixed mind-set. It led to students who were afraid of difficult tasks, who were less resilient and less persistent when they made mistakes. They only cared about looking smart and never doing anything that would contradict this. Praising student's intelligence saddles students with a big liability.

What we should be praising is their ability to process, the strategies that they are using, the effort that they are putting into the work, the persistence they show, there focus in doing the problem. These are the things that you can do the next time you strike a problem and are struggling. This is how you can be successful in the future.

You can grow intelligence - the harder you work in mathematics the smarter you will get. When you work hard to solve problems your mathematics brain will grow. When you try your maths brains will grow.

Teachers and parents should stop and listen to the fixed mind-set messages we all have in our heads. The things they tell themselves about themselves, the things they tell themselves about students. Don't evaluate them, just listen to them. Then start talking back to them with growth mind-set messages. We need to be aware of them to change our talk and to change our teaching practice.

The sabbatical challenged and changed my thinking on the teaching of mathematics. It was an amazing journey and the wonderful thing for me is that I have a colleague at school and together we are challenging staff thinking and moving our school into the 21st century. The goal is to have children's learning and achievement improve through the simple premise of them liking and understanding what mathematics is. Mathematics at our school must be about understanding, not performing

Singapore Schools

The majority of the schools in Singapore are state controlled and vary from 1200 1500 students. The schools that I visited head between 1250 and 1450 students attending. This meant there were approximately 80 to 90 classes operating at six levels Primary 1 to 6.

The composition of the classrooms are as follows Primary one and two had up to 30 students per class and Primary 3 to 6 had 40 and above.

School begins at 7:30 AM and continues to 1:30 PM. On most days following those there are core curriculum assistance times or supplementary classes and they run from 1:30 to 3:30pm. Classroom teachers are responsible and take these extra classes.

There is a quarter hour break and this is staggered so Primary three and five have theirs at 9 o'clock, Primary four and six at 9:30 and Primary one and two at 10 AM. Lunch two is staggered and lasts for half an hour.

School fees - local students pay $11.50-$12 50 per student per month while foreign student fees work out at around $3000 per year. The government puts into every child's account $250 per year to cover the school fees and books etc.

Staff receive between one and two hours non-contact time per day but have to pick up the supplementary hours after the school day finishes at 1:30. During this non contact time specialist teachers come and take class - for example for music and PE and social studies. Each teacher must take up to 3 core subjects and the government is moving to make this only two. The three are maths English and science and all other subjects are taken by specialists.

Parental support is very very high in Singapore especially in the core subjects where parents purchase supplementary texts for students to work on at home and this is above the two hours of regular homework per night. They also pay for extra tuition for their children.

The reason for this is high level of out of school ‘learning and tuition’ is the extremely important Primary Six end of year examination which determines the high school that the student then goes on to, which ultimately affects the lifestyle of the child and the parent care in the future.

If progress is not made by students then teachers provide extra time above supplementary time and also extra homework and tuition. Students are expected to do at least 2 hours of homework each night set from school then the extra that parents provide via their supplementary texts or extra tuition classes.

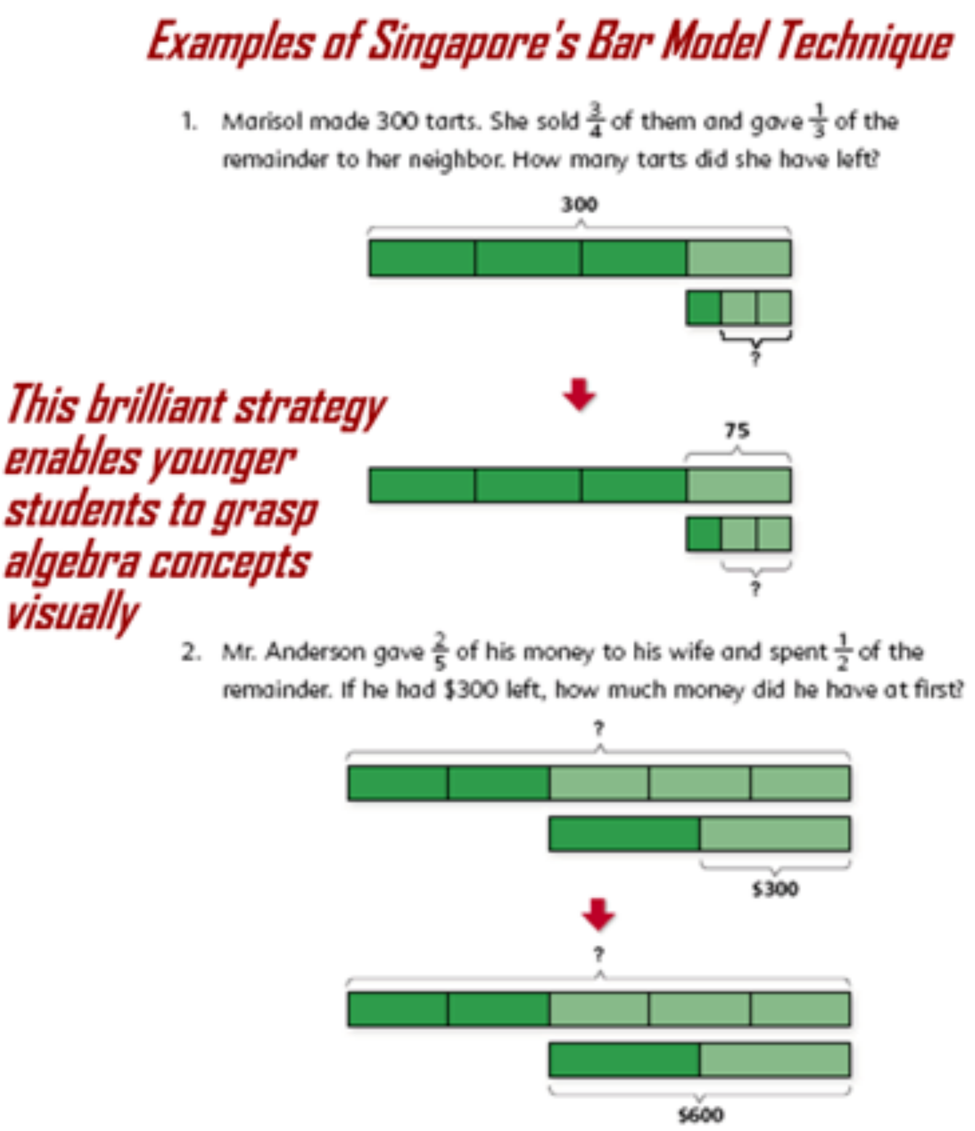
Mathematics in Singapore

Singapore mathematics is based heavily on the ideas of Jerome Brunner. It advocates the concrete, pictorial and abstract approach. In this approach students are provided with the necessary learning experiences and meaningful context using concrete manipulative and pictorial representation to help them learn abstract mathematics.

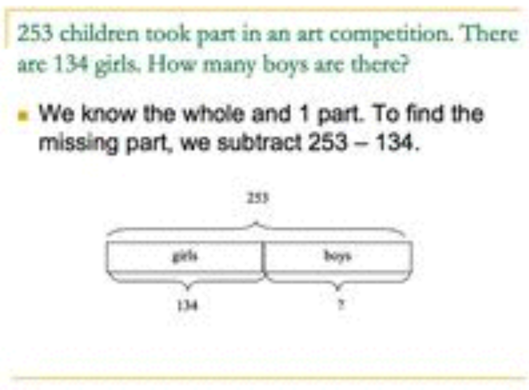
The Model Method for problem-solving as it is commonly known in Singapore was an innovation in the teaching and learning of mathematics to address the issue of students having difficulty with word problems.

This approach entails students drawing a pictorial model to represent mathematical qualities (known and unknown) and their relationships (part-whole and comparison) given in a problem to help them visualise and solve the problem.

The models are an essential element of the concrete-pictorial-abstract approach. Students make use of apparatus (manipulatives) to make sense of the part-whole and comparison concepts. They progress to drawing of rectangular bars as pictorial



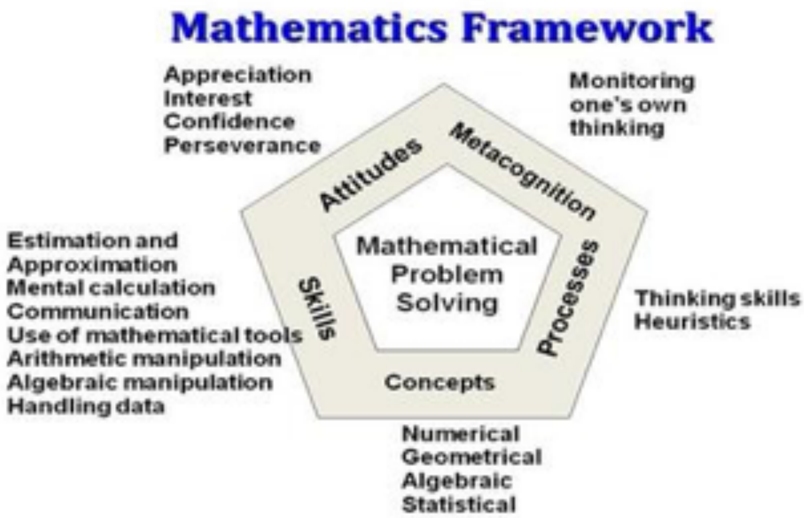
An Example of the Part Whole Model Strategy



representations of the models and use the models to help solve abstract mathematical word problems.

The Model Method shows clearly the problem structure and the known and unknown factors involved in the problem. It provides a visual tool that enables students to determine which operation to use to solve the problem.

The Singapore mathematics curriculum is guided by the mathematics framework introduced in 1990 to develop students mathematical abilities in particular their



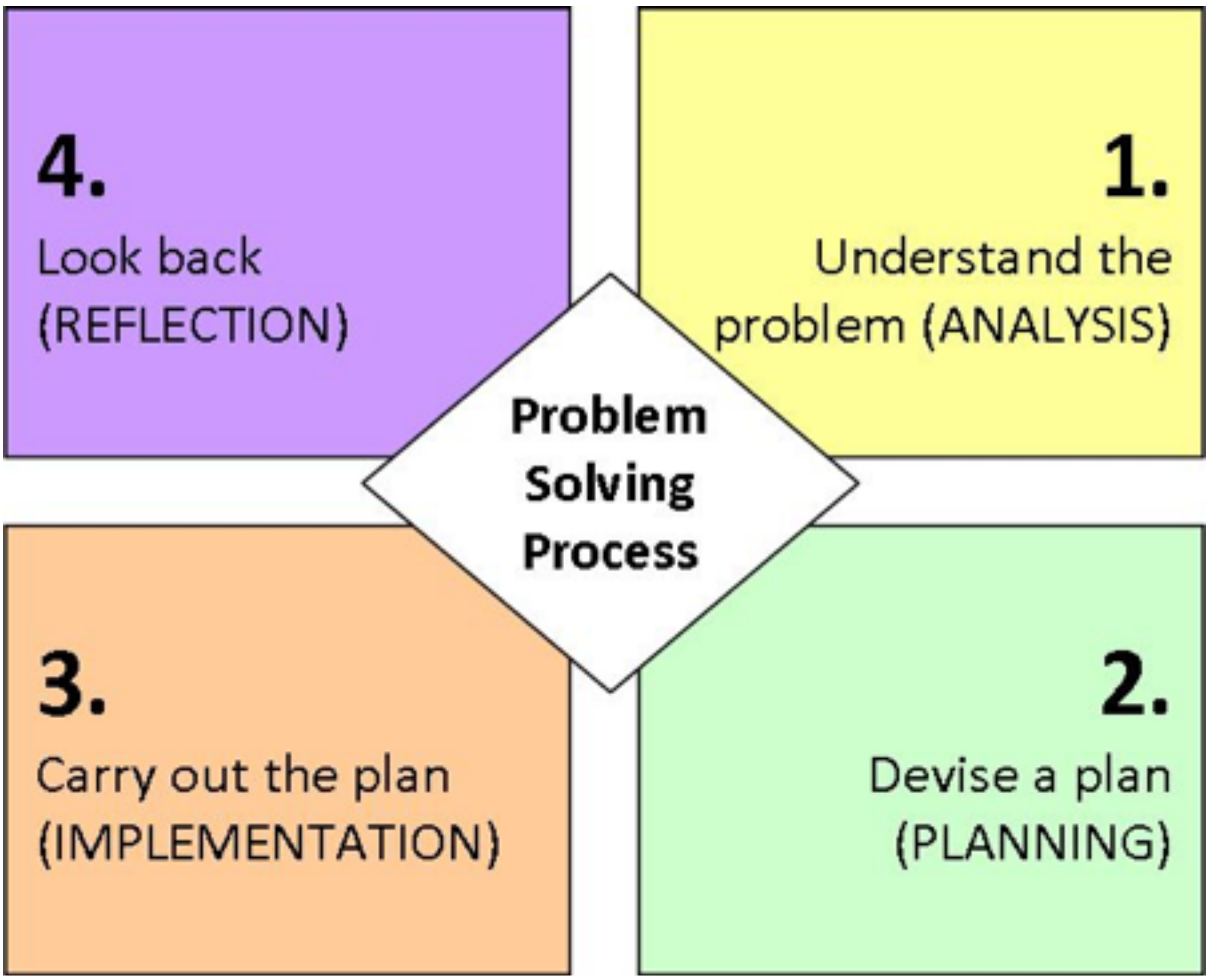
problem solving abilities. The mathematics framework shows the underlying principles of an effective mathematics program that is applicable to all levels from primary to advanced levels. It sets the direction for the teaching learning and assessment of mathematics.

Mathematical problem solving is central to mathematics learning. It involves the acquisition and application of mathematics concepts and skills in a wide range of situations. The development of mathematical problem solving ability is dependent on five inter-related components. These are

- concepts
- skills
- processes
- attitudes
- metacognition.

A new word for many of us is 'heuristics.' A heuristic is an Ancient Greek: εὑρίσκω, "find" or "discover") word and is any approach to problem solving, learning, or discovery that employs a practical method. Heuristics are what students do to approach a problem when the solution is not obvious. Example of heuristics are:

- Give a representation - draw a diagram, make a list, use equations
- Make a calculated guess - guess and check, look for patterns, make suppositions
- Go through the process - act it out, work backwards, before-after; and
- Change the problem - restate the problem, simplify the problem, solve part of it.



George Polya's Problem Solving

George Polya was a Hungarian mathematician who emigrated to the United States. In 1945 he published a book called 'How to Solve it' where he outlined a successful way to tackle mathematical problems.

Polya advocated a number of steps that one goes through to solve mathematical problems as the diagram above shows.

1. Analyse - understand the problem

The first step of Polya's Process is to Understand the Problem. Some ways to tell if you really understand what is being asked is to:

- State the problem in your own words.
- What exactly what is being asked.
- Identify the unknowns.
- Figure out what the problem tells you is important.
- Identify any information that is irrelevant to the problem.

2. Devise a Plan

Now that we understand the problem, we have to Devise a Plan to solve the problem. We could:

- Look for a pattern
- Review similar problems
- Make a table, diagram or chart
- Write an equation
- Use guessing and checking
- Work backwards
- Identify a sub-goal

3. Implement - carry out the plan

The third step in the process is the next logical step: Carry Out the Plan. When you carry out the plan, you should keep a record of your steps as you implement your strategy from step 2.

4. Reflect - check your

The final step in the process is very important, but many students skip it, feeling like they have an answer so they can move on now. The final step is to Look Back, which really means to check your work.

- Does the answer make sense?
- Sometimes you can add wrong, or multiply when you should have divided, then your answer comes out clearly wrong if you just stop and think about
- Check your result.
- Checking your result could mean solving the problem in another way to make sure you come out with the same answer.

And that is all there is to Polya's 4-Step Process to Problem Solving:

Much emphasis in Singapore is placed on problem solving and using the model method to visualise what is involved in the problem and what operation is needed to solve it. It is a step that we in New Zealand have missed out and one that urgently needs to be taught and included.